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Fast and accurate pseudoinverse with sparse matrix reordering and incremental approach

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Outline

- **Introduction**
- Proposed Method
- Experiment
- Conclusion

Research Question

- **Q. How can we compute the pseudoinverse of a sparse feature matrix efficiently and accurately?**
  - For solving machine learning problems
  - Pseudoinverse is a **generalized inverse** for all types of matrices
    - Plays a crucial role in **obtaining best-fit solutions** to the linear systems
    - Various applications in machine learning domain

Problem Definition

- **Pseudoinverse of a sparse matrix**
  - Moore-Penrose Inverse via low-rank SVD
  - **Inputs**
    - A **sparse** matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ and a target rank r
  - **Output**
    - MP Inverse $\mathbf{A}^{\dagger} \simeq \mathbf{V}_{n \times r} \mathbf{\Sigma}_{r \times r} \mathbf{U}_{r \times m}^{\top}$
      - $\mathbf{A}$ is decomposed into $\mathbf{U\Sigma V}^{\top}$ via SVD
      - If r is the full rank, the equality holds.
      - Otherwise, it is a best approximate $\mathbf{A}^{\dagger}$ for rank r
  - **Target Application:** Multi-label Linear Regression

Limitations

- **Previous methods have high costs for computing Pseudoinverse**
  - *Especially for relatively large rank r*
    - Needed for high accuracy
  - **SVD:** $O(mn^2)$ & **Randomized-SVD:** $O(mr^2)$
  - **Krylov** sub-space method & **frPCA** for spares matrices
    - Effective for very low rank r
- **C. How to efficiently compute it without loss of accuracy?**

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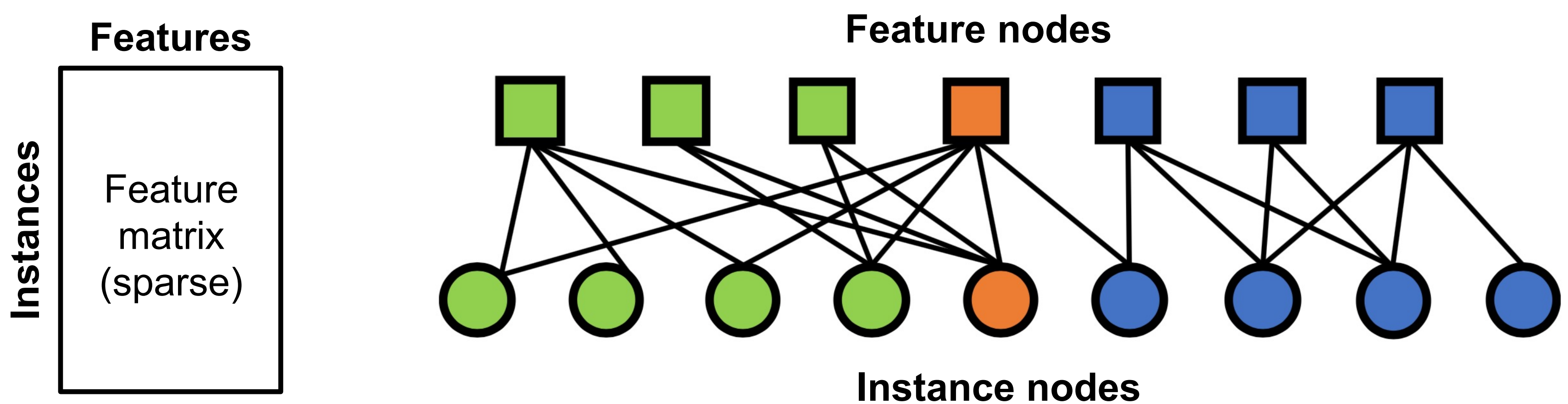
Proposed Method

- FastPI **(Fast PseudoInverse)**
  - **Novel**, **fast**, and **accurate** method for approximate pseudoinverse for spare matrices
- **Ideas**
  - **Idea 1)** Many feature matrices are highly sparse and skewed
    - Can be reordered such that their non-zeros are concentrated
  - **Idea 2)** The SVD of a large and sparse block diagonal sub-matrix is easy-to-compute
  - **Idea 3)** The final SVD is efficiently obtained by an incremental update method

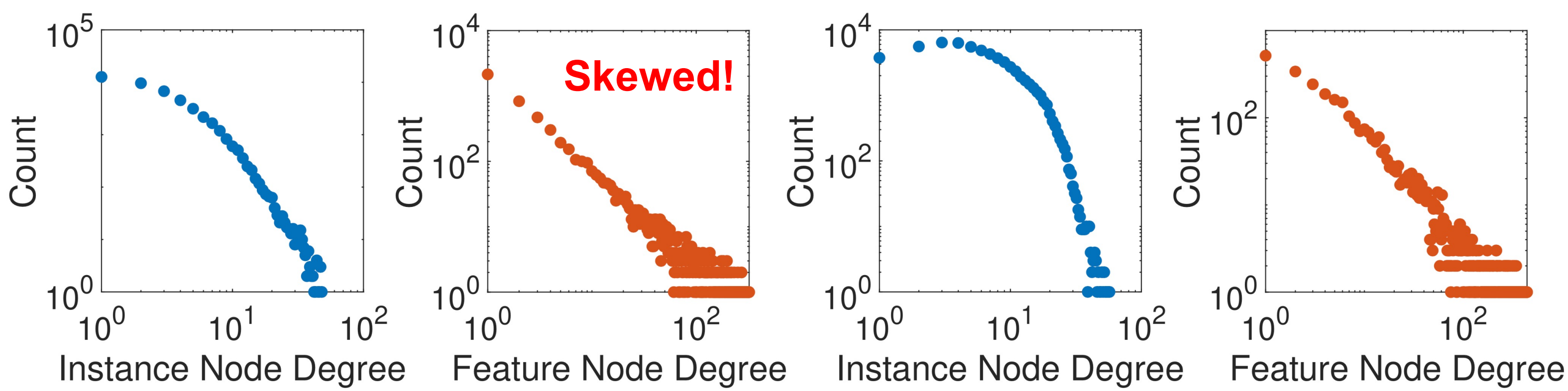
Sparse Feature Matrices

- **Sparse feature matrices are considered as bipartite networks**
  - Instance nodes to feature nodes



Sparse Feature Matrices

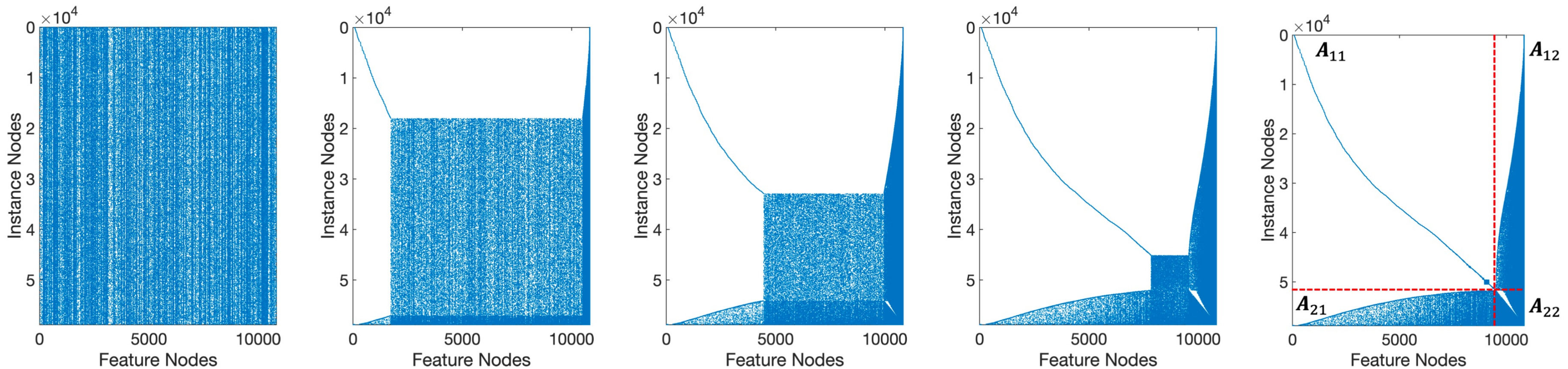
- **Degree distributions of bipartite networks from real-world feature matrices are highly skewed!**



(a) The Amazon dataset (b) The RCV dataset

Sparse Matrix Reordering

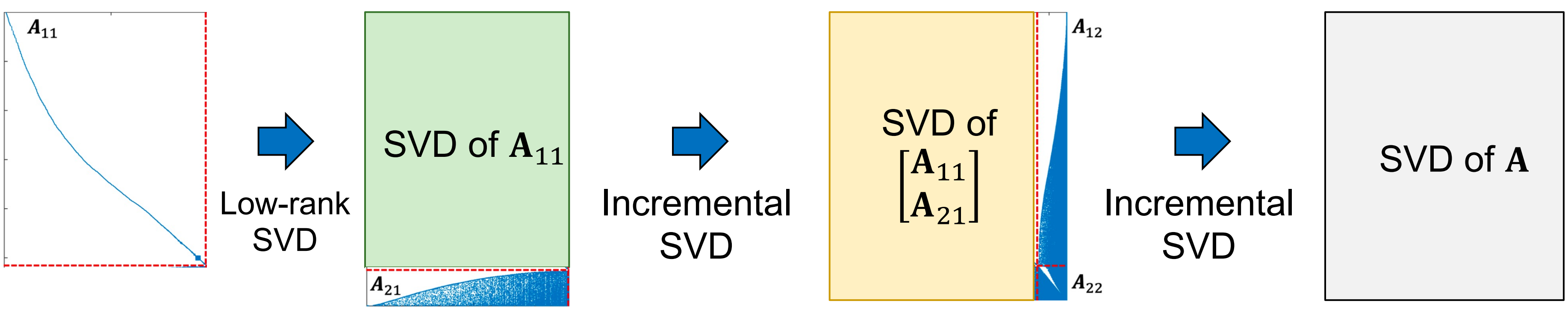
- **Sparse rectangular matrix can be reordered as follows:**
  - See the paper for details
  - The non-zero entries are concentrated



(a) The original matrix (b) After thirty iterations (c) After eighty iterations (d) After 115 iterations (e) After the final (119) iteration

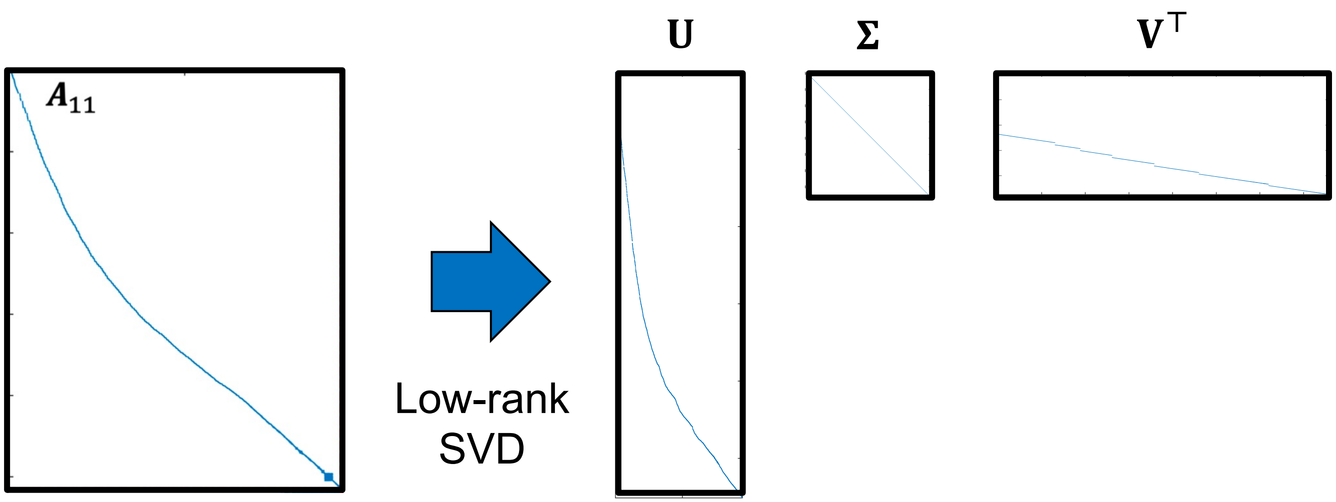
SVD on Reordered Matrix

- **How can we compute SVD of the reordered matrix while exploiting the sparsity?**
  - **Step 1.** Compute the SVD of $\mathbf{A}_{11}$
  - **Step 2.** Incrementally update it for $\mathbf{A}_{21}$
  - **Step 3.** Incrementally update it for $\begin{bmatrix}\mathbf{A}_{12} \\ \mathbf{A}_{22}\end{bmatrix}$



SVD on Reordered Matrix

- **SVD of the large sparse rectangular diagonal submatrix $\mathbf{A}_{11}$**
  - Easy-to-compute by computing SVD of each rectangular block & the results are also sparse!
  - Reason that we can accelerate the speed!



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Experimental Setting

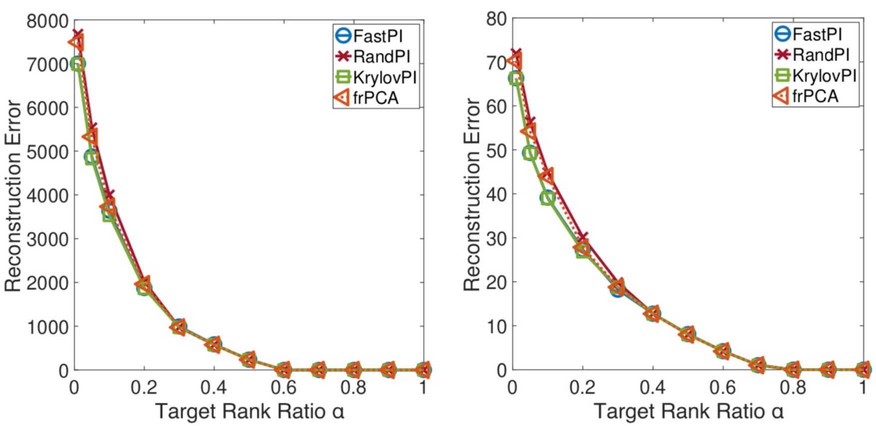
- **Experimental Questions**
  - Q1. Reconstruction error
  - Q2. Accuracy of multi-label linear regression
  - Q3. Efficiency

- **Datasets:** 4 real-world feature matrices
  - Amazon, RCV, Erulex, Bibtex

- **Methods**
  - **FastPI (proposed)**, RandPI, KrylovPI, frPCA

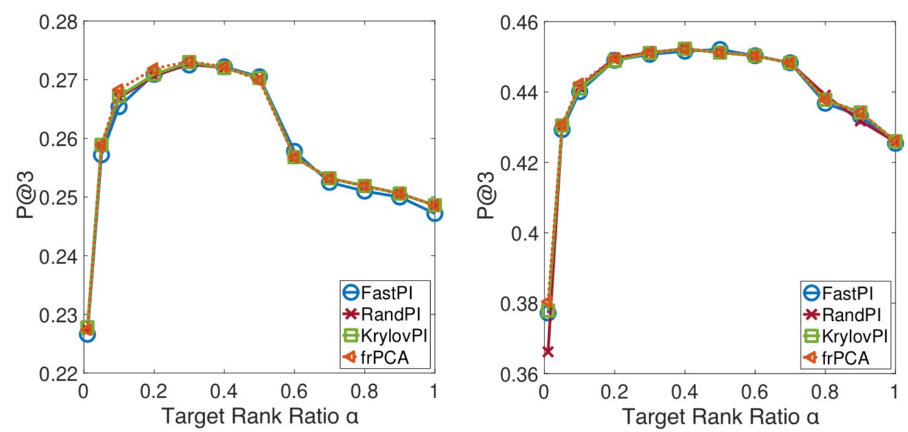
Reconstruction Error & Accuracy

- **FastPI produces similar reconstruction error & accuracy to other methods**
  - Can compute Pseudoinverse without loss of accuracy



(a) Amazon (b) RCV

Reconstruction Error

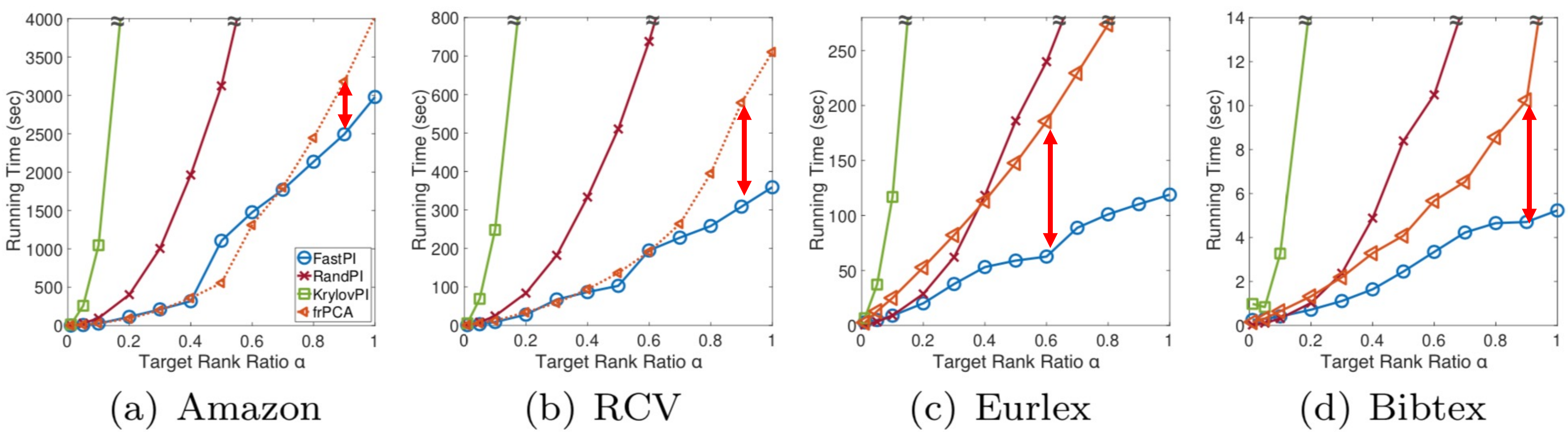


(a) Amazon (b) RCV

Accuracy of Multi-label Linear Regression

Efficiency

- FastPI is faster than other methods
  - Especially for relatively large rank!
  - However, they are similar when the rank is small ⇒ Need to improve this as future work



Running Time

Conclusion

- FastPI **(Fast PseudoInverse)**
  - **Idea 1)** Many feature matrices are sparse and skewed
  - **Idea 2)** The SVD of a sparse block diagonal matrix is easy-to-compute
  - **Idea 3)** The final SVD is efficiently obtained by an incremental update method
- **Experimental Results**
  - **FastPI** computes the approximate pseudoInverse of a sparse matrix **without loss of accuracy**
  - **FastPI** is **faster** than other competitors for **relatively large rank**

Thank you!

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